Solution by the PROPOSER.

Let I, J be the positions of Ike and Jim at the start; L, M their respective positions at any subsequent time t.

Let m, n be the coordinates of L; p, q the coordinates of M; ϕ the angle LM makes with IJ, $a=IJ$.

Then $dm/dt=v\cos\phi$, $dn/dt=v\sin\phi$, $p=m+a\cos\phi$, $q=n+a\sin\phi$.

$V^2=(dp/dt)^2+(dq/dt)^2$

$=(dm/dt-a\sin\phi\, d\phi/dt)^2+(dn/dt+a\cos\phi\, d\phi/dt)^2$

$=(v\cos\phi-a\sin\phi\, d\phi/dt)^2+(v\sin\phi+a\cos\phi\, d\phi/dt)^2$

$=v^2+a^2(d\phi/dt)^2$; $\therefore d\phi/dt=\sqrt{(V^2-v^2)}/a=b$.

$\therefore \phi=bt$, since $\phi=0$, when $t=0$.

$\therefore dm/dt=v\cos bt$, where $b=\sqrt{(V^2-v^2)}/a$.

$\therefore m=-\dfrac{v}{b}\sin bt$, $n=\dfrac{v}{b}(\cos bt-1)$. Therefore, Ike describes a circle.

Also, $p=a\cos bt-(v/b)\sin bt$, $q=a\sin bt+(v/b)(\cos bt-1)$.

Therefore, Jim also describes a circle.

MECHANICS.

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191. Proposed by DR. L. E. DICKSON, The University of Chicago.

Give the axiomatic principle of Physics which is equivalent to the theorem on the compound of two circles ("Graphical Methods in Trigonometry," MONTHLY, June-July, 1905).

Solution by G. B. M. ZERR, A. M., Ph. D., 4243 Girard Avenue, Philadelphia, Pa.

There are two principles that might be considered equivalent to the theorem on the compound of two circles. First, the parallelogram of velocities; second, the parallelogram of forces.

These might be named the compound of two velocities and the compound of two forces. We can state both under one theorem as follows:

The compound of a $\left\{\begin{matrix}\text{velocity}\\ \text{force}\end{matrix}\right\}$ OP with a $\left\{\begin{matrix}\text{velocity}\\ \text{force}\end{matrix}\right\}$ OR is the diagonal OQ of the parallelogram $OPQR$.

The proof by vectors follows at once. Regarding OP, OR, OQ as vectors we get at once $OP+OR=OQ$.

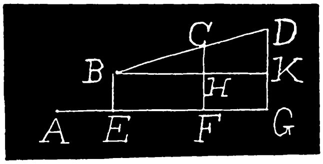
207. Proposed by W. J. GREENSTREET, M. A., Marling School, Stroud, England.

A portion of a parabola is bounded by the curve, the axis and an ordinate. A circle is inscribed to the figure which is regarded as a plane lamina. The area of the inscribed circle is now punched out. Find the centroid of what is left.

Solution by G. B. M. ZERR, A. M., Ph. D., 4243 Girard Avenue, Philahelphia, Pa.

Let $y^2=4ax$ be the parabola, b the abscissa of the portion considered,

r the radius of the circle. Then $(x-b+r)^2+(y-r)^2=r^2$ is the equation to the circle. Hence the invariants are $\Delta=-4a^2$, $\Delta'=-r^2$, $\theta=-4a(b-r+a)$, $\theta'=-4a(b-r)$. In order that the circle and parabola may touch $(\theta\theta'-9\Delta\Delta')^2=4(\theta^2-3\Delta\theta')\times(\theta'^2-3\Delta'\theta)$.



$\therefore 16r^5+(13a-48b)r^4+8(a^2+6b^2-14ab)r^3+8(15ab^2-2b^3-9a^2b)r^2+32ab(3ab-a^2-2b^2)r+16ab^2(b^2+a^2-2ab)=0$. This gives the value of r.

Area of semi-parabola $=\frac{4}{3}b\sqrt{(ab)}$, area of circle $=\pi r^2$, area of portion left $=\frac{4}{3}b\sqrt{(ab)}-\pi r^2$.

Let A be the vertex of the parabola, B the centroid required, C the centroid of the semi-parabola, D the centroid of the circle. Let (α, β) be the coordinates of B. The coordinates of C are $(\frac{3}{5}b, \frac{3}{4}\sqrt{ab})$; of D, $(b-r, r)$.

$$\frac{CD}{BC}=\frac{HK}{BH}=\frac{GF}{EF}=\frac{\frac{4}{3}b\sqrt{(ab)}-\pi r^2}{\pi r^2} \text{ or } \frac{EG}{EF}=\frac{4b\sqrt{(ab)}}{3\pi r^2}.$$

$$\therefore \frac{b-r-\alpha}{\frac{3}{5}b-\alpha}=\frac{4b\sqrt{(ab)}}{3\pi r^2}. \quad \therefore \alpha=\frac{3[4b^2\sqrt{(ab)}+5\pi r^3-5\pi br^2]}{5[4b\sqrt{(ab)}-3\pi r^2]}.$$

$$\text{And } \frac{DK}{CH}=\frac{EG}{EF}=\frac{4b\sqrt{(ab)}}{3\pi r^2}=\frac{r-\beta}{\frac{3}{4}\sqrt{(ab)}-\beta}; \quad \therefore \beta=\frac{3(ab^2-\pi r^3)}{4b\sqrt{(ab)}-3\pi r^2}.$$

208. Proposed by W. J. GREENSTREET, M. A., Marling School, Stroud, Eng.

Hanging at rest over a smooth pulley are two equal scale pans of the same mass. Two equal particles, the one inelastic and the other elastic, are simultaneously dropped from the same height one into each scale pan. Show that each impact after the first must occur when the pans have returned to the *status quo ante*, and find the total space described by either pan before motion ceases.

Remark by G. B. M. ZERR, A. M., Ph. D., 4243 Girard Avenue, Philadelphia, Pa.

This is the same as problem 121, Mechanics. A solution of this problem is found in Vol. VIII, No. 10, pp. 203-4, October, 1901.

NUMBER THEORY AND DIOPHANTINE ANALYSIS.

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141. Proposed by PROF. R. D. CARMICHAEL, Anniston, Ala.

Given that the highest factor of a prime p contained in $m!$ is p^{m-s}; find general expressions involving p and m and s, from which, when a solution is possible, m can be determined when s is a given integer and p is a given prime. Is it then possible in any case to have more solutions than one?